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Establishment of the Riemannian Structure of Space-Time by Classical Means

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Efforts at providing a physical-axiomatic foundation of the space-time structure of the general theory of relativity have led, when based on simple empirical facts about freely falling particles and light signals, in a satisfying manner only to a Weyl space-time. By adding postulates based on quantum theory, however, the usual pseudo-Riemannian space-time can be reached. We present a new *classical* postulate which provides the same results. It is based upon the notion of the radar distance between freely falling particles and demands the approximate equality of the growth of the radar distance for particle pairs of equal, small initial velocities. We show that given this, a property results, as found in earlier work by the author, that distinguishes between Weyl and Lorentz space-times. The property refers to a special metric and decides whether its metric connection has the given free-fall worldlines as geodesics or not. It consists in the vanishing of the mixed spatiotemporal components g_{i4} of this metric in suitable coordinates along the worldline of the freely falling observer, as the rest system of which the coordinates are constructed.

1. INTRODUCTION

1.1. Introduction

Almost all work in the general theory of relativity starts with some mathematical assumptions about the space-time model in which the investigation is to be pursued. Although there are of course motivations in the background, one should not accept such assumptions without asking for sound physical substantiation. In classical general relativity the common framework of the space-time models used in most cases is a Lorentzian (or pseudo-Riemannian) manifold (M, g) with a metric tensor g of signature $(\pm, \pm, \pm, \mp)$; but sometimes other or generalized space-times are considered, i.e., this

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picture is not self-evident enough to exclude all alternatives. Thus, many authors have tried to formulate physical foundations for the mathematical structures existing in the space-time models. For instance, the topology of the used manifolds participates in determining which physical effects can occur, so it is interesting to see if one can give a physical basis for it.

A typical way of providing physical foundations for space-time geometry is to consider a small number of simple physical objects and simple experiments with them, to translate them into mathematical axioms, and to derive that space-time has to be seen as a Lorentz manifold; we will sometimes call such reasoning a space-time theory (STT). Typical primitive concepts for this purpose are freely falling physical systems and light signals; statements about these do not presuppose much physics. The most influential of such theories and a kind of paradigm is that of Ehlers *et al.* (1972) (EPS), which has stimulated a lot of further work; but surprisingly, those postulates which directly express the properties of freely falling particles and light rays yield only a Weyl instead of a Lorentz manifold. Formally a Weyl geometry is distinguished from the latter by having a conformal equivalence class in place of a single metric, and in general none of the conformal metrics has the free-fall worldlines as the geodesics of its metric connection. Physically the crucial difference is the possibility of a so-called "second clock effect," that is, the path or history dependence of the clock rates. That free fall of particlelike bodies and light propagation yield only a Weyl geometry has revived interest in Weylian space-time models (for instance, Perlick, 1991; Chandra, 1984; Köhler, 1978) and led to the presumption that quantum theoretic considerations might be necessary to close the gap to the usual general relativistic geometry (Ehlers, 1973). Audretsch (1983) found a way to accomplish this (see also Audretsch *et al.*, 1984); later a comprehensive axiomatic approach applying matter wave fields as primitive objects was developed by Audretsch and Lämmerzahl (1988, 1991a,b, 1995).

Another recent STT (Schröter, 1988; Schröter and Schelb, 1992, 1993; Schelb, 1992) has motivated a new look at this problem. It is a theory which applies in part similar concepts to EPS, i.e., purely classical means, and is of interest here in particular because it tries vigorously to avoid all superfluous and possibly confusing, not directly physically motivated mathematics. In this theory one gets relatively naturally the Lorentzian space-time structure; a comparable "Weyl conundrum" does not appear. Thus the question is raised whether for the EPS theory there is an alternative to the recourse to quantum theory in order to achieve its pseudo-Riemannian completion.

As a result of the considerations which this new theory has inspired (cf. also Schelb, 1995b), we present here a new postulate, which excludes Weyl manifolds as EPS models. It involves only light signals and freely falling particles in formulating in an obvious way the concepts "radar distance" and "radar velocity." Roughly, the postulate expresses that a freely falling observer measures for a pair of freely falling particles ejected by him the same radar distance, at least if they have small initial radar velocities. Its deeper meaning is the formulation of a property of the path structure of free-fall worldlines, which to our knowledge has not been previously achieved.

1.2. Mathematical Basis

Our mathematical point of departure is determined by the axioms of EPS up to Axiom C, i.e., by a Weyl spacetime. Mathematically, a Weyl manifold is a triple $(M, \mathcal{G}, \nabla^W)$ consisting of:

(i) A four-dimensional manifold M, which we assume to be of class C^∞, the topology of which is Hausdorff and second countable.
(ii) A conformal equivalence class $\mathcal{G}$ of metrics g on M with Lorentzian signature $(+1, +1, +1, -1)$.
(iii) A symmetric linear connection ∇^W on M with vanishing torsion.

The set of timelike geodesics of ∇^W (without specification of parametrization) constitute the so-called *projective structure* of $(M, \mathcal{G}, \nabla^W)$; they are physically interpreted as the worldlines of freely falling particles. The null paths of $\mathcal{G}$ are interpreted as worldlines of light signals; they are ∇^W-geodesics, too. The Axiom C of "compatibility" of the conformal and projective structures expresses the physical experience that freely falling particles can approximate ("chase") the worldlines of light arbitrarily closely.

A Weyl manifold where there is a $g \in \mathcal{G}$, so that for its metric connection one has $\nabla^g = \nabla^W$, is called *reducible*.

1.3. Outline of This Approach

The strategy of this paper is as follows: We formulate a new postulate by which we will enrich the sketched mathematical structures of the EPS theory. The models of the EPS theory (i.e., Weyl space-times) which additionally obey this postulate constitute a subclass of the EPS space-times. Our goal is to show that this subclass consists of *reducible* Weyl manifolds. In another paper (Schelb, 1995a) we have shown that reducible Weyl manifolds (which "contain" a Lorentz manifold) are distinguished from irreducible ones by a certain property. Thus the task here is to demonstrate that the models of the subclass possess this property. The property relates to a special metric out of $\mathcal{G}$ and to its representation in suitable locally geodesic coordinates; it consists in the vanishing of the g_{i4} components, $i = 1, 2, 3$, of the metric along a certain particle worldline. Although this may seem a somewhat hidden feature, it is nonetheless a natural one from a heuristic point of view: One can reverse the direction of thought and look back from a special model to the axiomatics. If in our case one takes a Lorentzian manifold and reconstructs in it the EPS construction, then the choices which we make are near at hand. Also, the quoted more recent STT serves as a heuristic guide in this.

However, in contrast to this condition in special coordinates, the postulate from which it is to be derived should be formulated in a coordinate-independent manner; this is done. Furthermore, it is based upon no more than the primitive notions of the EPS axiomatics, essentially free fall and light propagation—that is, purely classical concepts.

In Section 2 we introduce the necessary tools and state the announced postulate. The purpose of Section 3 is to enable the formulation of the mentioned condition: We give a brief sketch of the construction of the conformal class of the EPS theory, choose a special metric from it, and construct the special coordinate system. Sections 4 and 5 draw the consequences of the postulate in these coordinates: The former shows in which way the pairs of particles used in the postulate appear in the coordinates, the latter evaluates its proper statement with respect to the metric components.

2. RADAR DISTANCES AND THE POSTULATE

In this section we introduce the tools for the statement of the postulate. Of central importance is the concept of "radar measurements." By this we mean a procedure where the observer (a particle) emits a light signal which is then reflected in an event on the worldline of another particle in its neighborhood and arrives again at the observer worldline. If there is a parametrization ("clock") at the latter, two numbers can be assigned to the events of emission and rearrival, respectively, of the light signal on it. The difference of these numbers may be seen as a "radar distance."

With regard to the question of which kind of empirical experiment should be used as the basis of an STT, we take the view that such radar measurements are not only suitable, but are the most convincing kind of data for this. The above-quoted recent STT tries to formulate all its empirical inputs as statements about a version of such radar measurements.

Perlick (1987, 1994) in his work on the introduction of the concept of a standard clock in EPS models, respectively Weyl space-times, used similar tools. Since here we work in the EPS framework, we can essentially apply the terms which he developed.

Consider the worldline P of a particle together with a parametrization $\gamma: I \subset \mathbb{R} \mapsto P \subset M$, making it into a *curve*. According to Axiom L_1 of EPS there are for each event $q \in P$ two open neighborhoods $U \subset V \subset M$ so that any event $p \in U$ can be connected to P by exactly two light signals which do not leave V (see Fig. 1). Thus there are two events $q_1, q_2 \in P$ where the light signal starts and rearrives, respectively; if $\gamma(t_1) = q_1$ and $\gamma(t_2) = q_2$, the real numbers t_1 and t_2, with $t_2 > t_1$, can be used to define the functions $\rho: U \mapsto \mathbb{R}^+\setminus\{0\}$ and $\tau: U \mapsto I \subset \mathbb{R}$ by $\rho(q) := \frac{1}{2}(t_2 - t_1)$, $\tau(q) := \frac{1}{2}(t_2 + t_1)$. If there is a worldline P' of another particle passing through U, the observer P can apply this to all events of $P' \cap U$; by using τ as parametrization of P', he can define (and measure) $\rho(\tau)$ for P'. But one can also see the radar distance ρ as a function of the parametrization of P, since $\tau \in (t_1, t_2)$: $\rho(t) := \rho(\tau)$ for $\tau = t$. It is in this form, as a function of the parameter t of P, that it will be used in the following.

So far, we have considered arbitrary, not necessarily freely falling particles; all the "radar" concepts work for this general class of particles. In the sequel, however, we restrict our attention to freely falling particles, for the observer P as well as for the "test particle" P'. Furthermore, we will consider only situations where P and P' coincide at one event; intuitively this can be seen as an event where P' "starts moving away" from P.

Having registrated a radar distance to P' as function of its parameter t, P can also determine the rate of change of the distance, the "radar velocity" (likewise a function of t): $\rho'(t) = d\rho/dt$. Since the function $\rho(t)$ is not differenti-

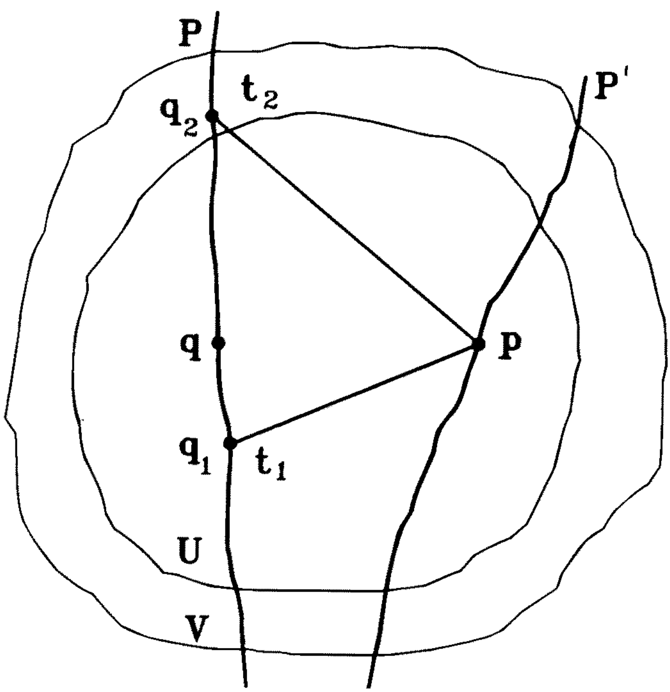

Fig. 1. Situation of radar measurements.

able on P, one can in the event of intersection p_0 only consider the limit of $\rho'(t)$ from one side; we are interested in the limit from above, i.e., $t > t_0$:

$$\lim_{t \to t_0} \rho'(t) =: \beta \tag{1}$$

β can be read as an initial velocity with which P' moves away from P in p_0. Although $\rho(t)$ and $\rho'(t)$ are defined via a special parametrization, the limit value β is independent of the parameter of P; this can be shown directly by methods similar to those used below or, more quickly, by inspection of certain coordinate-free expressions involving β which are derived in Perlick (1987). Though the parametrization independence is gratifying, it is not crucial for what follows.

Let us now consider situations in which another freely falling particle P'' coincides with P and P' in the same p_0. Now, P can measure radar distances and velocities of P'', too, and compare with those of P'. Let us assume, in particular, that P' and P'' move away in p_0 "in opposite directions"; with the available mathematical concepts such an expression makes sense: It means that the tangent vectors to the curves P, P', P'' in $T_{p_0}M$ are linearly dependent. If we designate one direction with a $+$, the other with $-$, the distances of P' and P'' can be denoted as $\rho^+(t)$ and $\rho^-(t)$, respectively. We will consider now pairs of freely falling particles P', P'' moving away from P in a p_0 with equal initial velocities: $\beta(P') = \beta(P'')$. Our question is: How will the radar distances $\rho(t)$ develop further with t for such a pair? Will they remain nearly equal, as is true in classical mechanics in the case of free motion and equal initial velocity, or not? The former means that radar signals which are sent out in the same event to P' and P'' rearrive at P in nearly the same event. The "nearly" means "at least for pairs of sufficiently small β"; i.e., we consider pairs of equal β and go to the limit $\beta \to 0$. By the postulate we distinguish among the EPS models those in which this approximate equality is given everywhere.

As argument under the limit we use $\rho(t_1)$ instead of the above $\rho(t)$, where t_1 belongs to the event of emission $\gamma(t_1) \in P$ of the radar signals; in order to avoid confusion we denote the distance as a function of the time of emission by $r(t_1)$. This is physically more natural (one compares measurements started in the same event) and makes no mathematical difference for our statements. By abuse of notation we omit the subscript "1" in the following. The expression $r^+(t)/r^-(t)$ in the postulate is the ratio of radar distances for one pair P', P'' of equal β, the measurement of which was initiated in the event $\gamma(t)$. We cannot assume that such radar measurements for particles ejected in a p_0 can be performed in *all* events on the observer worldline occurring "later" than p_0. But according to the above introduction of the radar method there is at least a finite "time interval" in which this is possible. In the postulate we will designate it as (t_0, t').

Postulate 1. For every particle P and every event $p_0 \in P$ the following statement holds independently of the parametrization: Consider the set of those pairs of freely falling particles whose worldlines run through p_0 in opposite directions with equal β with respect to P. If t_0 is the parameter value assigned to p_0, then there is a $t' > t_0$ so that

$$\lim_{\beta \to 0} \frac{r^+(t)}{r^-(t)} = 1 \qquad \text{for all} \quad t \in (t_0, t') \tag{2}$$

Remarks. 1. The postulate employs only *classical* concepts, and nothing beyond what is available in the original EPS theory. It is of purely *local* character.

2. The experimental testability and plausibility of the postulate is obvious: A freely falling satellite which ejects simultaneously freely falling test particles in opposite directions and with sufficiently small equal initial velocities can test the validity of the postulate. According to present knowledge, violations of it are not expected.

3. In the example of a Minkowski space-time the postulate is fulfilled even without the limit of small initial velocities.

3. TOOLS FOR APPLYING THE POSTULATE

3.1. EPS Conformal Structure

Below we will evaluate the consequences of the postulate for a certain metric in the EPS space-time models. As a basis for the selection of this special metric we have to recall the way in which a conformal structure is introduced in the EPS theory with the help of particles and the properties of light signals.

In a first step one defines an auxiliary function h_q: If $q \in M$ is an arbitrary event, and P an arbitrary particle worldline passing through q, then as described above there are to each $p \in U(q)\backslash P$ exactly two events e_1, e_2 $\in P$ connected to P by light signals. If there is a parametrization f of P, f: $\mathrm{R}^1 \mapsto P$, we can apply this in order to define in $U(q)$ a function h_q: $U(q) \mapsto \mathrm{R}$ by

$$h_q(p) := -\{f(e_2) - f(q)\} \cdot \{f(e_1) - f(q)\} \tag{3}$$

Since only differences are involved, a shifting of the parameter values so that $f(q) = 0$ does not change h_q, but gives it a simpler form:

$$h_q(p) = -f(e_2) \cdot f(e_1) \tag{4}$$

(we have not changed the notation of f). If, in particular, $p \in P$, there are no actual light signals from P to p: In this case we define $e_1 = p = e_2$ and thus can extend the definition of h_q to $U(q) \cup \overline{P}$ [where $\overline{P} = P \cap U(q)$]. For $p \in P$ one has

$$h_q(p) = -f^2(p) \tag{5}$$

Decisive for the suitability of the h_q-functions as a basis of the conformal class is its value in H_q, which is defined as set of all events which are connected with the given q by a (worldline of a) light signal: If $p \in H_q$, this means either $e_1 = q$ or $e_2 = q$, so one can read off from (3) or (4)

$$h_q(p) = 0 \tag{6}$$

From the postulates it follows [or, in another version of the theory, it itself is a postulate (Meister, 1992)] that h_q is of class C^2 with respect to its argument p. The differentiability allows us to define the metric tensor as a second derivative of h_q taken at the point q; i.e., the definition is point by point.

Definition. Given an event $q \in M$, a particle P, so that $q \in P$, and any parametrization f of P, then a metric tensor g_q is defined in q by the following prescription: For every pair of vectors Y_q, $Z_q \in T_qM$ (resp. extensions of Y_q and Z_q around q) the map g_q: $T_qM \times T_qM \mapsto \mathrm{R}$ yields the number

$$g_q(Y_q, Z_q) = Y(Z(h_q))|_q \tag{7}$$

Some properties have to be proven in order to demonstrate the well-definedness of g_q:

(i) In general such a twofold derivation of a function is not a tensor; in this case, however, it is because of

$$dh_q|_q = 0 \tag{8}$$

(ii) $g_q(Y_q, Z_q) = g_q(Z_q, Y_q)$.

(iii) If $V_q \in T_qM$ is the tangent vector of the worldline of a light signal (i.e., along this worldline $h_q \equiv 0$), then

$$g_q(V_q, V_q) = 0 \tag{9}$$

(iv) The auxiliary functions h_q depend on q; in each q, the procedure gives another function. Correspondingly, so far one does not know anything about the connection of g_{q_1} and g_{q_2} for two neighboring events q_1, q_2. Thus one postulates (one way or the other; cf. the remark above on C^2 for h_q): g_q is of class C^2 with respect to q.

In the definition of g_q a special particle P and a special parameter f have been used, but one can show without difficulty that a change of f or the transition to another particle $\tilde{P}$ passing through q produces only a conformal factor. In obvious symbolic notation

$$\tilde{g}_q[\tilde{P}, \tilde{f}] = e^{\phi(q)} g_q[P, f] \qquad \text{with} \quad \phi(q) \in \mathbb{R}^1 \tag{10}$$

So as the result of these constructions one gets a conformal structure $\mathcal{G}$ on M.

3.2. Choice of a Metric

If, however, one is interested in a specific single metric g, then from the preceding a way to construct one explicitly is immediate: One removes the arbitrariness of P and f, but chooses a specific representative of both. Concretely, we choose an event p_0 and a freely falling particle P with $p_0 \in P$. Let us use the notation γ for a curve of this particle, γ: $I \subset \mathbb{R} \mapsto P \subset M$. The curve γ is a geodesic curve of ∇^W; thus we can choose in particular an affine parameter t of γ; if its corresponding tangent vector is denoted as $\dot{\gamma}$, then $\nabla^W_{\dot{\gamma}}\dot{\gamma} = 0$. Given γ and t, we have for each $q \in P$ a function h_q and thus along γ, resp. P, a specified metric g. Affine parametrizations are determined only up to a *constant* factor; but since a change of this factor multiplies the metric g_q likewise only with a constant number, it is not necessary and not legitimate to make an explicit choice for it.

If one looks back at (5), one sees the following property of any of the EPS metrics: If g_q is given by the choice of a particle P and of a parametrization f, then, if X_q is the tangent vector to P with respect to f in q,

$$g_q(X_q, X_q) = -2 \tag{11}$$

Thus, as a matter of convenience we complete our definition of a distinguished metric by the prescription

$$\tilde{g} := \tfrac{1}{2} g \tag{12}$$

In what follows we work with $\tilde{g}$ and thus for notational convenience omit the tilde.

3.3. Coordinates

As the next step we introduce a special coordinate system, the so-called locally geodesic or Riemannian normal coordinates. [A heuristic motivation of this choice—and of that of the metric—is that it brings about a partially comparable situation to our background theory (Schröter and Schelb, 1992; 1993).] Because of the exponential map $TM \mapsto M$ they can be introduced in a neighborhood of any event in a manifold with a linear connection. Their essential characteristic is that all geodesics through a certain event appear in the coordinates as straight lines through the origin. That has the consequence that the connection coefficients in these coordinates vanish at the origin; but this does not play a role in our considerations.

For the construction of the coordinates we have to choose a point p_0 (for the origin), an affine parameter on each geodesic, and in $T_{p_0}M$ a basis $E_j, j = 1, 2, 3, 4$. We specify them as follows (ψ denotes the coordinate map $U \subset M \mapsto \mathbb{R}^4$):

(i) Let p_0 be any point of γ, i.e., on the worldline of the freely falling particle used in the definition of g above. Thus $\psi(p_0) = (0, 0, 0, 0)$.

(ii) If $\dot{\gamma}$ denotes the tangent vector with respect to the affine parameter used in the definition of g, let us choose the basis $E_1, \ldots, E_4$ in $T_{p_0}M$ so that $E_4 = \dot{\gamma}$, and use this affine parameter for the construction of the coordinates. Thus $\psi(\gamma(t)) = (0, 0, 0, t)$.

(iii) The other three elements of the basis are chosen as having some properties with respect to the g defined above in p_0: Let E_1, E_2, E_3 be spacelike and such that $g_{p0}(E_i, E_4) = 0$, $i = 1, 2, 3$, and $g_{p0}(E_i, E_j) = 0$, $i, j = 1, 2, 3$, $i \neq j$.

(iv) The affine parameters of the worldlines $\tilde{\gamma}_i$ corresponding to E_i, i.e., with $\dot{\tilde{\gamma}} = E_i$, $i = 1, 2, 3$, are such that

$$g_{p0}(\dot{\gamma}, \dot{\gamma}) = -g_{p0}(E_i, E_i), \qquad i = 1, 2, 3 \tag{13}$$

Analogously, for the timelike geodesic worldlines $\hat{\gamma}$ through p_0,

$$g_{p0}(\dot{\gamma}, \dot{\gamma}) = g_{p0}(\dot{\hat{\gamma}}, \dot{\hat{\gamma}}) \tag{14}$$

We remark that there are physical reasons for the use of these conditions which we do not discuss here (Schröter and Schelb, 1993) and that these conditions (14) can be used to extend the definition of g beyond the single curve γ.

Obviously for null geodesics the affine parameter cannot be fixed in this way; but it has to be such that further particle worldlines crossing the null curves (thus not passing through p_0) have continuous coordinate curves. This determines the parameters of the null geodesics uniquely.

What do we know about our special g in these coordinates?

(a) Because of its role in the construction of the coordinates one has at the origin

$$g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(1, 1, 1, -1) \tag{15}$$

(b) For the chosen γ and its parameter we have $g(\dot{\gamma}, \dot{\gamma}) = -1$. If the coordinates of the tangent vector $\dot{\gamma}$ are denoted by v^μ, then we know that $g_{\mu\nu}v^\mu v^\nu = -1$; but by construction v is along γ, i.e., on the four-axis of the coordinates, of the form $v = (0, 0, 0, 1)$, so that this reduces to

$$g_{44} = -1 \tag{16}$$

along the four-axis.

4. SYMMETRICITY OF PAIRS OF EQUAL β

The purpose of this section is to demonstrate how a pair of particles of equal initial velocity β appears in the constructed locally geodesic coordinates. It is presupposed that the observer which measures the β is mapped on the four-axis of the coordinates; since the observed particles are freely falling, their coordinate curves are straight lines. Evidently we can restrict ourselves to the consideration of pairs whose worldlines lie in the plane spanned by the one- and four-axes of the coordinates. Let us denote $P(1, 4) := \{x = (x^1, x^2, x^3, x^4) | x^2 = x^3 = 0\}$. Any other pair can be rotated into $P(1, 4)$ by appropriate choice of the basis vector E_1.

We will demonstrate that any pair of equal β in $P(1, 4)$ is symmetric with respect to the four-axis. If we denote the curves of the pair by γ_+ and γ_-, respectively, and parametrize them as prescribed in (14), and denote the corresponding tangent vectors by u_+ and u_-, then, by assumption, $u^j_+ = u^j_- = 0$ for $j = 2, 3$ and $u^4_+ = u^4_-$. The task in this section is the proof of the following result.

Proposition 1. $\beta(\gamma_+) = \beta(\gamma_-)$ means $u^1_+ = -u^1_-$.

In order to show this, we consider first a very simplified situation where the proposition is obvious and then check that in the general situation u_+ and u_- do not differ from those in the special situation.

The simplifying assumption is that in the coordinates the metric components are constant: $g_{\mu\nu} \equiv \eta_{\mu\nu}$. In the general case this equality holds only at the origin; but since our statement involves limits at the origin, we can trace this back to the Minkowski situation.

We will write the entities in the general case by a tilde, to distinguish them from those in the special case: $\tilde{\rho}$ vs. ρ, $\tilde{t}$ vs. t, and so on.

Proof. (a) If $g_{\mu\nu} \equiv \eta_{\mu\nu}$ throughout the coordinate system, then the worldlines of the radar signals are straight lines with the usual angle of $\pi/4$ against the axes. The radar distance $\rho(t)$ of any event $(x^1, 0, 0, x^4)$ on $\gamma_\pm$ is determined by $\rho = |x^1|$, $t = x^4$; since $\gamma_\pm$ are straight lines, $d\rho/dt = \text{const} = \beta$.

(b) In the general case, $g_{\mu\nu}(x) \neq g_{\mu\nu}(0)$, the worldlines of light differ in general from straight lines. This means that if one considers the same coordinate lines $\gamma_\pm$ and the same event $(x^1, 0, 0, x^4)$ on $\gamma_\pm$ as in (a), then the events on the four-axis where a light signal to $(x^1, 0, 0, x^4)$ starts and rearrives will differ from the corresponding ones in (a). Hence the resulting values of $\tilde{\rho}$ and $\tilde{t}$ will be different from ρ and t (although they all tend to zero on approaching the origin); cf. Fig. 2. We will show that

$$\tilde{\beta}(\gamma_\pm) := \lim_{\tilde{t}\to 0} \tilde{\rho}'(\tilde{t}) = \lim_{t\to 0} \rho'(t) = \beta(\gamma_\pm) \tag{17}$$

If this is valid together with (a), the proposition follows.

(b1) We will use the fact that to any event $(x^1, 0, 0, x^4)$ on $\gamma_\pm$ a time $\tilde{t}$ as well as a t as in (a) is uniquely assigned. Thus we can read $\tilde{t}$ as a function of t which is such that $\tilde{t}(0) = 0$. The limits $\tilde{t} \to 0$ and $t \to 0$ are those in which $(x^1, 0, 0, x^4)$ approaches the origin on $\gamma_\pm$. Thus

$$\lim_{\tilde{t}\to 0} \tilde{\rho}'(\tilde{t}) = \lim_{t\to 0} \tilde{\rho}'(\tilde{t}(t)) \tag{18}$$

Therewith we can rewrite (17)

$$\frac{\lim_{t\to 0} \tilde{\rho}'(\tilde{t}(t))}{\lim_{t\to 0} \rho'(t)} = \lim_{t\to 0} \frac{\tilde{\rho}'(\tilde{t})}{\rho'(t)} = \lim_{t\to 0} \frac{d\tilde{\rho}/dt \cdot dt/d\tilde{t}}{\rho'(t)} = \lim_{t\to 0} \frac{\tilde{\rho}(\tilde{t}(t))}{\rho(t)} \cdot \lim_{t\to 0} \frac{dt}{d\tilde{t}} \tag{19}$$

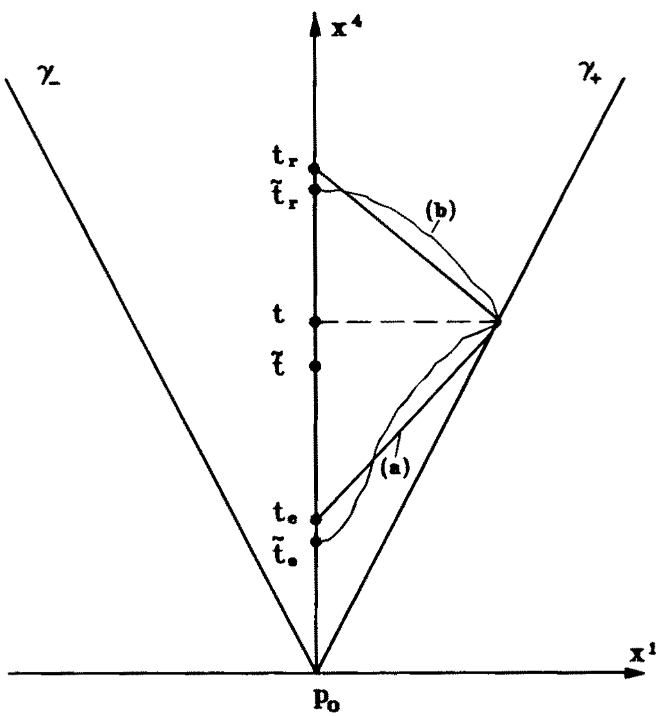

Fig. 2. Two particles of equal β in the coordinates.

where in the last step the rule of de l'Hôspital was applied. If both factors on the right-hand side are unity, (17) is true.

In the following an event on $\gamma_\pm$ is characterized by its x^4 coordinate, which is identical to the t in situation (a). Let us denote the corresponding events of radar emission and reception on the four-axis for (a) and (b) by t_e, t_r, $\tilde{t}_e$, $\tilde{t}_r$, respectively, which thus all are treated as functions of t. Recall the definitions $\tilde{\rho}(\tilde{t}) = \frac{1}{2}(\tilde{t}_r - \tilde{t}_e)$, $\rho(t) = \frac{1}{2}(t_r - t_e)$, $\tilde{t} = \frac{1}{2}(\tilde{t}_r + \tilde{t}_e)$, $t = \frac{1}{2}(t_r + t_e)$. In the first instance we assume that the worldlines of these radar signals lie in $P(1, 4)$.

(b2) Here we will show

$$\lim_{t\to 0} \frac{\tilde{t}_e}{t_e} = 1 \tag{20}$$

Let us designate by $\tilde{\sigma}$ the coordinate curves of the radar signals and by λ their parameter, and let $\tilde{v} = d\tilde{\sigma}/d\lambda$ be its tangent vector. The parametrization is such that in the "target" on $\gamma_\pm$ one has $\tilde{\sigma}(\lambda_0) = (x^1, 0, 0, x^4)$, and so that $|\tilde{\sigma}^1(\lambda)| \equiv \lambda$, and thus $|\tilde{v}^1| \equiv 1$. The last sentence of (b1) means $\tilde{v}^2 = \tilde{v}^3 = 0$.

For $\tilde{v}$, we have $g(\tilde{v}, \tilde{v}) = 0$; solved for $\tilde{v}^4$, this gives

$$\tilde{v}^4 = \left\{\left(\frac{g_{14}}{g_{44}}\right)^2 - \frac{g_{11}}{g_{44}}\right\}^{1/2} \pm \frac{g_{14}}{g_{44}} \tag{21}$$

which we will use shortly.

We apply $\tilde{\sigma}$ and $\tilde{v}$ to express $\tilde{t}_e(t)$ explicitly. For any target on $\gamma_\pm$ we have

$$\tilde{t}_e + \int_0^{\lambda_0} \tilde{v}^4(\lambda)\, d\lambda = t \tag{22}$$

In the situation of (a) the analogue for the same target is

$$t = t_e + \int_0^{\lambda_0} v^4(\lambda)\, d\lambda = t_e + \lambda_0 \tag{23}$$

since here $\sigma(\lambda)$ is straight and $v^4(\lambda) = \text{const}$. Equation (21) expresses how $\tilde{v}^4$ depends on the coordinates x, viz., via $g_{\mu\nu} = g_{\mu\nu}(x)$. The latter are in the EPS theory at least of class C^2 and $g_{44} \neq 0$; thus we can develop

$$\tilde{v}^4(x) = \tilde{v}^4(0) + x^1 \cdot \frac{\partial \tilde{v}^4}{\partial x^1}(0) + x^4 \cdot \frac{\partial \tilde{v}^4}{\partial x^4}(0) + \cdots \tag{24}$$

where $\tilde{v}^4(0) = v^4(0) = 1$. Applying this to the values of $\tilde{v}^4$ along the curve $\tilde{\sigma}(\lambda)$, we can write (22) [with c_i abbreviating the constants $(\partial\tilde{v}^4/\partial x^i)(0)$, $i = 1, 4$] as

$$\tilde{t}_e = t - \int_0^{\lambda_0} (1 + c_1 x^1(\lambda) + c_2 x^4(\lambda) + \cdots)\, d\lambda \tag{25}$$

Since we are interested in the limit $t \to 0$, it is enough to consider the $\tilde{v}^4$ in a small neighborhood of the origin, say $U_\epsilon(p_0)$. Since there we have $x^1(\lambda) < \epsilon$, $x^4(\lambda) < \epsilon$ for all $\lambda \in (0, \lambda_0)$, one can estimate

$$\int_0^{\lambda_0} c_1 x^1(\lambda)\, d\lambda \leq c_1 \lambda_0 \epsilon, \qquad \int_0^{\lambda_0} c_2 x^4(\lambda)\, d\lambda \leq c_2 \lambda_0 \epsilon \tag{26}$$

In the limit process $t \to 0$ the light curves and vectors $\tilde{v}(\lambda)$ are changed; but near enough to the origin the development (24) and thus the estimate (26) are true for all of them. For decreasing ϵ also the parameter λ_0 for the target on $\gamma_\pm$ goes to zero with the same order; so all terms except the first in the integral in (25) go at least quadratically in ϵ to zero in our limit process; we have

$$\frac{\tilde{t}_e}{t_e} = \frac{1}{t - \lambda_0}\left(t - \lambda_0 - \int_0^{\lambda_0} c_1 x^1\, d\lambda - \int_0^{\lambda_0} c_2 x^4\, d\lambda - \cdots\right) \tag{27}$$

Since the denominators $(t - \lambda_0) = t_e$ go only linearly with ϵ to zero, we have

$$\lim_{t \to 0} \frac{\tilde{t}_e}{t_e} = \frac{t - \lambda_0}{t - \lambda_0} = 1 \tag{28}$$

(b3) The same reasoning can be repeated for the reception times $\tilde{t}_r$ and t_r, with

$$\tilde{t}_r = t + \int_0^{\lambda_0} \tilde{v}^4(\lambda)\, d\lambda \tag{29}$$

and with the $\pm$ in (21) changing into $\mp$. Thus we can infer analogously

$$\lim_{t \to 0} \frac{\tilde{t}_r}{t_r} = 1 \tag{30}$$

(b4) Here we draw from (b2) and (b3) the consequence

$$\lim_{t \to 0} \frac{\frac{1}{2}(\tilde{t}_r + \tilde{t}_e)}{\frac{1}{2}(t_r + t_e)} = 1$$

(we omit in this calculation the subscript $t \to 0$). We have

$$\lim \frac{\tilde{t}_r + \tilde{t}_e}{t_r + t_e} = \left[\lim \frac{t_r + t_e}{\tilde{t}_r}\right]^{-1} + \left[\lim \frac{t_r + t_e}{\tilde{t}_e}\right]^{-1}$$

$$= \left[1 + \lim \frac{t_e}{\tilde{t}_r}\right]^{-1} + \left[\lim \frac{t_r}{\tilde{t}_e} + 1\right]^{-1} \qquad \text{because of (28) and (29)}$$

$$= \left[\lim\left(\frac{\tilde{t}_r}{\tilde{t}_r} + \frac{t_e}{\tilde{t}_r}\right)\right]^{-1} + \left[\lim\left(\frac{t_r}{\tilde{t}_e} + \frac{\tilde{t}_e}{\tilde{t}_e}\right)\right]^{-1}$$

$$= \lim\left(\frac{\tilde{t}_r}{\tilde{t}_r + t_e}\right) + \lim\left(\frac{\tilde{t}_e}{t_r + \tilde{t}_e}\right) \tag{31}$$

From $\lim(t_r/\tilde{t}_r)\cdot\lim(t_e/\tilde{t}_e) = 1\cdot 1$ it follows that $\lim \tilde{t}_r = \lim(t_r\cdot t_e/\tilde{t}_e)$, so that we can continue (31) by

$$= \lim\left(\frac{t_e\cdot t_r/\tilde{t}_e}{t_e\cdot t_r/\tilde{t}_e + t_e}\right) + \lim\left(\frac{\tilde{t}_e}{t_r + \tilde{t}_e}\right)$$

$$= \lim\left(\frac{t_r}{t_r + \tilde{t}_e}\right) + \lim\left(\frac{\tilde{t}_e}{t_r + \tilde{t}_e}\right)$$

$$= \lim\left(\frac{t_r + \tilde{t}_e}{t_r + \tilde{t}_e}\right) = 1 \tag{32}$$

Thus we have

$$\lim_{t\to 0} \frac{\tilde{t}(t)}{t} = 1 \tag{33}$$

This means that the second factor on the right-hand side of (19) is unity.

(b5) Completely analogously, one calculates on the basis of (b2) and (b3) for the remaining factor in (19)

$$\lim_{t\to 0} \frac{\tilde{\rho}(\tilde{t})}{\rho(t)} = \lim_{t\to 0} \frac{\frac{1}{2}(\tilde{t}_r - \tilde{t}_e)}{\frac{1}{2}(t_r - t_e)} = 1 \tag{34}$$

(b6) If we abandon the assumption that the light signals to $\gamma_\pm$ run inside $P(1, 4)$, how does this change the preceding considerations? In the development (24) appear additionally

$$x^i\cdot\frac{\partial\tilde{v}^4}{\partial x^i}(0), \qquad i = 2, 3$$

which, however, do not contribute in the limit $t \to 0$. The zeroth-order term $\tilde{v}^4(0)$ is, without the restriction to $P(1, 4)$, not uniquely determined; for a light curve emitted on the four-axis in a direction outside $P(1, 4)$ it might be different, say $\tilde{w}^4(0)$, to that inside $P(1, 4)$. But obviously in the limit $t \to 0$ these light curves have to approach $P(1, 4)$ again, since their target on $\gamma_\pm$ is in $P(1, 4)$, and thus in the limit the term $\tilde{v}^4(0)$ is the same as in the above considerations. Hence the proposition is proven in this case, too. ■

5. EVALUATION OF THE POSTULATE

In this section we will show that the validity of the postulate has the consequence that in the coordinate system of the observer γ for the chosen metric the components $g_{i4} = 0$, $i = 1, 2, 3$, along the four-axis. This is based upon the symmetricity of the pairs of particles of equal β derived just above.

Proposition 2. For an EPS space-time model which obeys the postulate, the following statement holds: The metric g constructed above has the property that in the locally geodesic coordinates of the freely falling particle γ its components g_{i4}, $i = 1, 2, 3$, vanish in the events $(0, 0, 0, x^4)$ for all $x^4 > 0$.

Our proof is similarly divided into two steps as in the preceding section. We consider two situations: (a) where in the neighborhood of $(0, 0, 0, t)$ the $g_{\mu\nu}$ have constantly the same values (in general different from $\eta_{\mu\nu}$) as in $(0, 0, 0, t)$, so that in this neighborhood the worldlines of light are straight lines; and (b) the general situation where $g_{\mu\nu}(x^1, x^2, x^3, x^4) \neq g_{\mu\nu}(0, 0, 0, t)$. The motivation for this is that the statement of the proposition refers only to $g_{\mu\nu}$ at the point $(0, 0, 0, t)$, so that we can trace things back to the simplified case.

We show that in the limit $\beta \to 0$ the rearrival times of radar signals emitted at $(0, 0, 0, t)$ to a particle become equal for both situations. Then the validity of the postulate has the consequence that the ratio of the rearrival times for a pair $\gamma_\pm$ of particles has to be unity, and from this then the statement about g_{i4} can be concluded.

We denote the particles of the pair of equal β by $\gamma_\pm$, respectively, and accordingly the corresponding distances and arrival times in the radar measurement by $r_\pm$, $t^\pm$. Since here we have always radar signals emitted at $(0, 0, 0, t)$, we will write for the time of emission throughout t, for the times of rearrival t_2. In case (a), where $g_{\mu\nu}(x^1, \ldots, x^4) \equiv g_{\mu\nu}(0, 0, 0, t)$, we will designate the arrival times with a caret: $\hat{t}_2^+$, $\hat{t}_2^-$ (see Fig. 3).

Lemma. The following condition holds:

$$\lim_{\beta\to 0} \frac{t_2^+}{\hat{t}_2^+} = 1 = \lim_{\beta\to 0} \frac{t_2^-}{\hat{t}_2^-} \tag{35}$$

Proof. Our reasoning is similar to that in Section 4. Because of the limit, we need only consider radar measurements taking place in the mentioned neighborhood of $(0, 0, 0, t)$.

In a first step we assume that the radar signals run inside $P(1, 4)$.

(i) The time of arrival of the radar signals to γ_+ on the four-axis is the sum of the time-of-flight away and back; hence

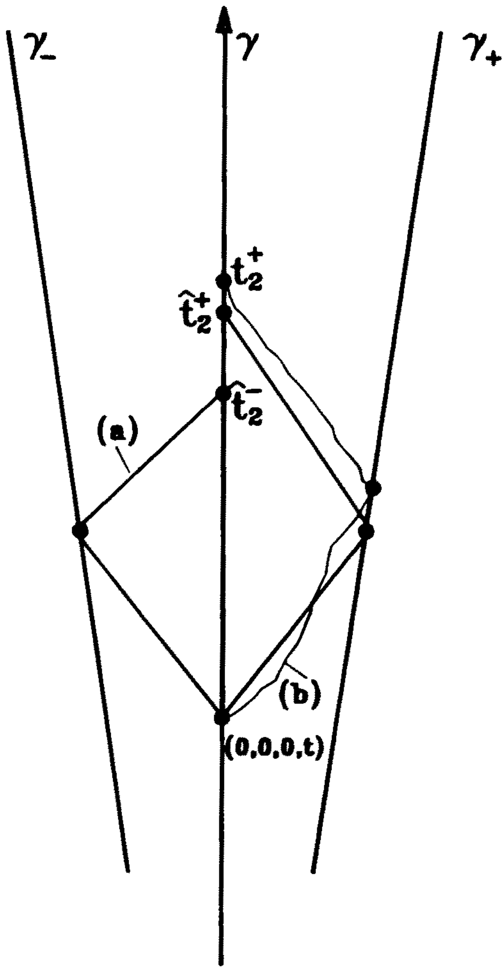

Fig. 3. Radar distance measurement for small β.

$$t_2^+ = t + \int_0^{\lambda_0} v_o^4(\lambda)\, d\lambda + \int_0^{\lambda_0} v_i^4(\lambda)\, d\lambda \tag{36}$$

where again the parameter λ is chosen as the x^1 coordinate, and v_o and v_i are the tangent vectors of the light worldline for the *o*utgoing and the *i*ncoming signals ($v_o^1 > 0$, $v_i^1 < 0$), respectively. Analogously, in the case $g_{\mu\nu}$ = const

$$\hat{t}_2^+ = t + \mu_0 \cdot \{w_o^4 + w_i^4\} \tag{37}$$

where μ designates the parameter and w_o, w_i are the tangent vectors of the light curves in this case. μ, like λ, is chosen as given by x^1, but since the light curves here in general will be different, so will be the events on γ_+, where they are reflected. Thus μ_0 and λ_0 might be different.

Because of the limit $\beta \to 0$ the relevant light curves lie inside a small neighborhood of $x_0 = (0, 0, 0, t)$; thus we develop [the parameter value $\lambda = 0$ belongs to $(0, 0, 0, t)$]

$$v^4_{o,i}(\lambda) = v^4_{o,i}(\lambda = 0) + \lambda \cdot \frac{dv^4_{o,i}}{d\lambda}(0) + \cdots \tag{38}$$

Abbreviating $C_{o,i} := (dv^4_{o,i}/d\lambda)(0)$, we know that

$$\left| \int_0^{\lambda_0} \lambda \cdot C_{o,i}\, d\lambda \right| = \left| C_{o,i} \cdot \frac{\lambda_0^2}{2} \right| \tag{39}$$

is small, since λ_0 is bounded by the smallness of the neighborhood of x_0. Then

$$\frac{t_2^+}{\hat{t}_2^+} = \frac{t + \lambda_0(v_o^4(0) + v_i^4(0))}{t + \mu_0(w_o^4 + w_i^4)} + \frac{\frac{1}{2}\lambda_0^2(C_o + C_i)}{t + \mu_0(w_o^4 + w_i^4)} + \cdots \tag{40}$$

μ_0 is of the same order as λ_0, so in the limit $\beta \to 0$ where λ_0 and μ_0 tend to zero, the second term and higher order terms on the right-hand side can be neglected. Thus

$$\lim_{\beta \to 0} \frac{t_2^+}{\hat{t}_2^+} = \lim_{\beta \to 0} \frac{t + \lambda_0(v_o^4(0) + v_i^4(0))}{t + \mu_0(w_o^4 + w_i^4)} \tag{41}$$

But according to their construction, $v^4_{o,i}(0) = w^4_{o,i}$, $g_{x_0}(v, v) = g_{x_0}(w, w) = 0$; so the remaining open question is to determine if $\lim_{\beta \to 0}(\lambda_0/\mu_0) = 1$.

(ii) Let p_1 and p_2 be the two events on the target particle γ_+ where the light signals in the cases $g_{\mu\nu} = \text{const}$ and $g_{\mu\nu} \neq \text{const}$ arrive, respectively; then $\lambda_0 = x^1(p_1)$, $\mu_0 = x^1(p_2)$. Now we can exploit the fact that p_1 and p_2 both lie on the same straight line γ_+ in the coordinates. Thus with a constant number $m \in \mathbf{R}$ we have

$$x^4(p_1) = m \cdot x^1(p_1), \qquad x^4(p_2) = m \cdot x^1(p_2) \tag{42}$$

In parallel to the preceding considerations [see (41)], we can write

$$\lim_{\beta \to 0} \frac{x^4(p_1)}{x^4(p_2)} = \lim_{\beta \to 0} \frac{t + \lambda_0 \cdot v_o^4(0)}{t + \mu_0 \cdot w_o^4} \tag{43}$$

Plugging (42) into this yields

$$\lim_{\beta \to 0} \frac{m \cdot \lambda_0}{m \cdot \mu_0} = \lim_{\beta \to 0} \frac{t + \lambda_0 \cdot v_o^4(0)}{t + \mu_0 \cdot w_o^4} \tag{44}$$

or

$$\lim_{\beta\to 0} \frac{\lambda_0}{\mu_0} = \lim_{\beta\to 0} \frac{t + \lambda_0 w_o^4}{t + \mu_0 w_o^4} \tag{45}$$

It follows that

$$\left[t + \lim_{\beta\to 0}(\mu_0 w_o^4)\right] \cdot \lim_{\beta\to 0} \frac{\lambda_0}{\mu_0} = t + \lim_{\beta\to 0}(\lambda_0 w_o^4) \tag{46}$$

and

$$t \cdot \lim_{\beta\to 0} \frac{\lambda_0}{\mu_0} + \lim_{\beta\to 0} \lambda_0 w_o^4 = t + \lim_{\beta\to 0}(\lambda_0 w_o^4) \tag{47}$$

This is possible only if

$$\lim_{\beta\to 0} \frac{\lambda_0}{\mu_0} = 1 \qquad \text{or} \qquad t = 0 \tag{48}$$

but here we have $t \neq 0$.

Thus from (i) and (ii) we can summarize:

$$\lim_{\beta\to 0} \frac{t_2^+}{\hat{t}_2^+} = 1 \tag{49}$$

Nothing is changed by exchanging γ_+ and γ_-; thus, likewise,

$$\lim_{\beta\to 0} \frac{t_2^-}{\hat{t}_2^-} = 1 \tag{50}$$

(iii) The generalizability to the case without the assumption that the light signals run inside $P(1, 4)$ can be concluded analogously to the previous section. ■

Proof of the Proposition. (a) From the lemma it follows that

$$1 = \lim_{\beta\to 0} \frac{t_2^+}{\hat{t}_2^+} \frac{\hat{t}_2^-}{t_2^-} = \lim_{\beta\to 0} \frac{t_2^+}{t_2^-} \lim_{\beta\to 0} \frac{\hat{t}_2^-}{\hat{t}_2^+} \tag{51}$$

The postulate states that $\lim_{\beta\to 0}[r_+(t)/r_-(t)] = 1$; because of the definition $r_\pm(t) = \frac{1}{2}(t_2^\pm - t)$, we know that $\lim_{\beta\to 0}[(t_2^+ - t)/(t_2^- - t)] = 1$ and thus $\lim_{\beta\to 0}(t_2^+/t_2^-) = 1$, since t is constant in the limiting process. Hence (51) means

$$\lim_{\beta\to 0} \frac{\hat{t}_2^-}{\hat{t}_2^+} = 1 \tag{52}$$

(b) Now we demonstrate that from (52) it results that in $x_0 = (0, 0, 0, t)$ we have $g_{14} = 0$. (Note that here the limit could be omitted since the $\hat{t}$ are times determined by straight worldlines of light.) The arrival times $\hat{t}_2$ are determined by [cf. (37)]

$$\hat{t}_2^+ = t + \mu_0\{w_o^4 + w_i^4\}, \qquad \hat{t}_2^- = t + \nu_0\{u_o^4 + u_i^4\} \tag{53}$$

The vectors u_o, u_i of the light to γ_- are such that $u_o = w_i$, $u_i = w_o$. Then it follows from (52) that $\mu_0 = \nu_0$; if $q_1 \in \gamma_+$ and $q_2 \in \gamma_-$ denote the events of the observed pairs of particles where the radar signals are reflected, this means that $|x^1(q_1)| = |x^1(q_2)|$, since the parametrizations μ and ν are made by the x^1 coordinate. The worldlines $\gamma_\pm$ are straight lines in $P(1, 4)$, hence $x^4(q_1) = x^4(q_2)$. Because we know that

$$x^4(q_1) = t + w_o^4 \cdot \mu_0, \qquad x^4(q_2) = t + w_i^4 \cdot \nu_0 \tag{54}$$

then necessarily $w_o^4 = w_i^4$. On the other hand, we have

$$w_o^4 = \left[\left(\frac{g_{14}}{g_{44}}\right)^2 - \frac{g_{11}}{g_{44}}\right]^{1/2} - \frac{g_{14}}{g_{44}} \tag{55}$$

$$w_i^4 = \left[\left(\frac{g_{14}}{g_{44}}\right)^2 - \frac{g_{11}}{g_{44}}\right]^{1/2} + \frac{g_{14}}{g_{44}} \tag{56}$$

where all the $g_{\mu\nu}$ are in $x_0 = (0, 0, 0, t)$; since $g_{44}(x_0) = -1$, $w_o^4 = w_i^4$ enforces $g_{14} = 0$ in $(0, 0, 0, t)$.

(c) By the use of pairs of particles $\gamma_\pm$ in $P(2, 4)$ and $P(3, 4)$ we get the same for g_{24} and g_{34}. ■

The reasoning so far has referred to one certain particle γ as "observer." But the entire procedure can be repeated with any other freely falling particle.

6. DISCUSSION

1. The event p_0 and the observing particle γ were arbitrarily chosen as the basis of our investigations. Obviously everything can be done analogously on the basis of any other event and particle. But is the g for which $g_{i4} = 0$ is found the same (or can it be made the same) for these different bases? That this is indeed the case is shown in Schelb (1995a).

2. The intuitive meaning of $g_{i4} = 0$ along the four-axis is that the light cone which at the origin of the coordinates is symmetric to the four-axis remains symmetric for at least a finite piece of the four-axis. In space-times with $g_{i4} \neq 0$, however, which are, as shown in Schelb (1995a), the irreducible Weyl space-times, the light cone starts tipping over outside the origin. Our locally geodesic coordinates represent the worldlines of freely falling particles for both a Weyl and a Lorentz space-time in a symmetrized way; the feature

by which the latter is distinguished from the more general former is that there this symmetricity is supplemented by an additional one with regard to the worldlines of light. Loosely speaking, Lorentz space-times, in comparison with Weylian ones, are characterized by a certain kind of symmetry between their conformal and projective structure.

3. The proofs here have been elaborated in coordinates, and hence are somewhat tedious. In order to replace them by more transparent coordinate-free arguments one might try to derive that a kind of Gauss lemma holds for the space-times obeying the new postulate, i.e., the conservation of orthogonality along geodesics. This idea is motivated through the observation that in Lorentz space-times, from the Gauss lemma it follows that $g_{i4} = 0$ (cf. Schelb, 1995a).

4. As indicated by the title of this paper, the basic physical meaning of our result is that quantum mechanical reasoning is not necessary for the establishment of the pseudo-Riemannian structure of space-time, since a postulate in simple classical terms provides the same result.

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